Statistical Analysis section

Two Samples: Tests on Two Means

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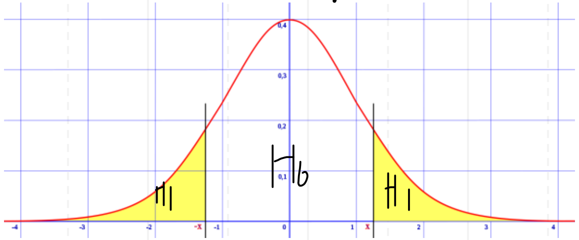
Two Samples: Tests on Two Means when variances known

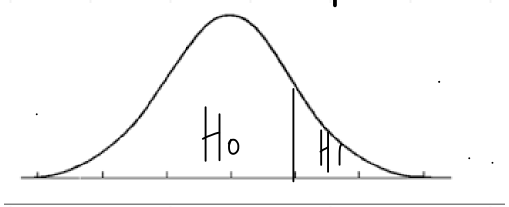
$\mu_1 - \mu_2 = d_0$	$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$; σ_1 and σ_2 known	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$z < -z_\alpha$ $z > z_\alpha$ $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$

$\mu_1 - \mu_2 \neq d_0$

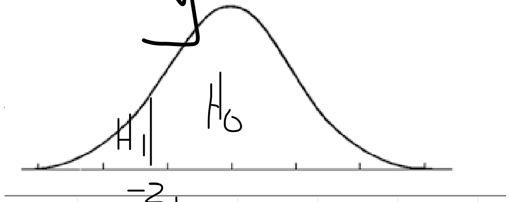


$-z_{\alpha/2}$ $z_{\alpha/2}$

$\mu_1 - \mu_2 > d_0$



z_α



$-z_\alpha$

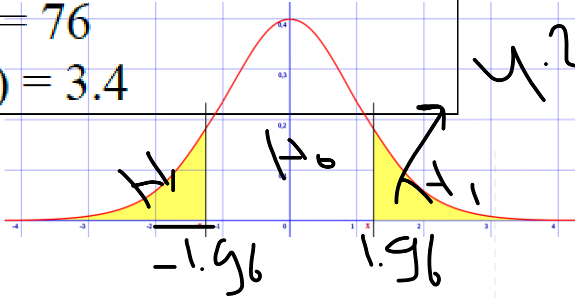
$\mu_1 - \mu_2 = 0$ $\quad \alpha/2 = 0.025$

Two Samples: Tests on Two Means when variances known

$\mu_1 - \mu_2 \neq 0$

10.30 Test $H_0: \mu 1 = \mu 2$ against $H_1: \mu 1 \neq \mu 2$. Use a 0.05 level of significance (α).

Sample 1	Sample 2
Sample size (n_1) = 25 Sample mean ($\overline{X}_1$) = 81 Population s.d. (σ_1) = 5.2	Sample size (n_2) = 36 Sample mean ($\overline{X}_2$) = 76 Population s.d. (σ_2) = 3.4



$$H_0: \mu_1 = \mu_2,$$
$$H_1: \mu_1 \neq \mu_2.$$

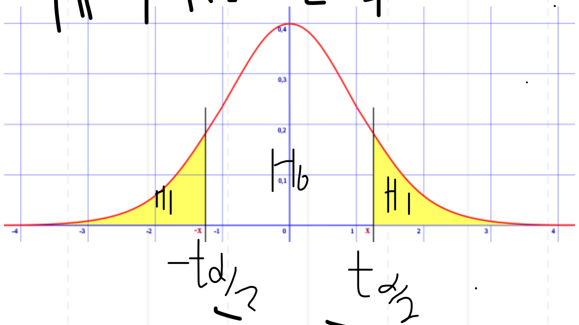
Since the variances are known, we obtain $z = \frac{81-76}{\sqrt{5.2^2/25+3.5^2/36}} = 4.22$. So, P-value≈ 0 and we conclude that $\mu_1 > \mu_2$.

Accept H_1

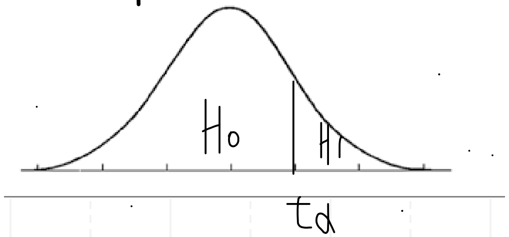
Two Samples: Tests on Two Means when variances unknown but equal

$\mu_1 - \mu_2 = d_0$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p\sqrt{1/n_1 + 1/n_2}};$ $v = n_1 + n_2 - 2,$ $\sigma_1 = \sigma_2$ but unknown, $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t < -t_\alpha$ $t > t_\alpha$ $t < -t_{\alpha/2}$ or $t > t_{\alpha/2}$

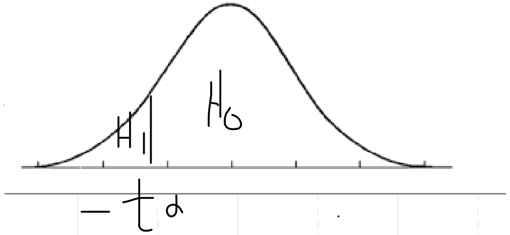
$H_1 \; \mu_1 - \mu_2 \neq d_0$



$\mu_1 - \mu_2 > 0$



$\mu_1 - \mu_2 < 0$



Two Samples: Tests on Two Means when variances unknown but equal

$\mu_2 - \mu_1 = 0$ $\mu_2 - \mu_1 < 0$

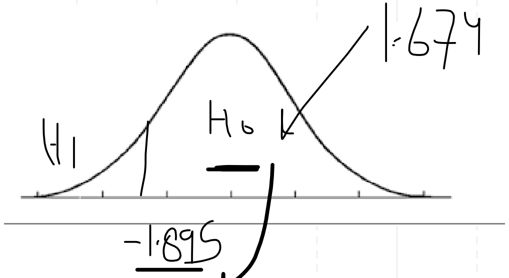
10.35 Test $H_0: \mu 2 = \mu 1$ against $H_1: \mu 2 < \mu 1$. Use a 0.05 level of significance (α).

Treatment	2.1	5.3	1.4	4.6	0.9
No Treatment	1.9	0.5	2.8	3.1	

Assume the populations to be approximately normal with **equal variances**.

$$H_0 : \mu_1 - \mu_2 = 0,$$
$$H_1 : \mu_1 - \mu_2 \le 0.$$

t_2 t_1



$\alpha = 0.05$

Critical region: $t < -1.895$ with 7 degrees of freedom.

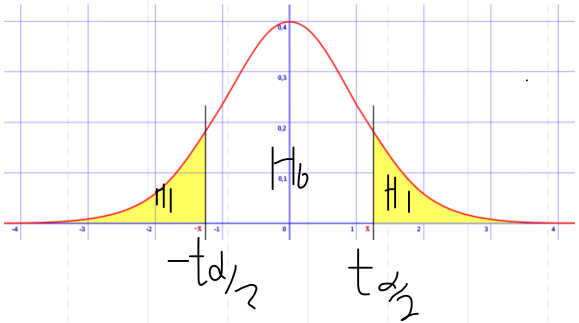
Computation: $s_p = \sqrt{\frac{(3)(1.363)+(4)(3.883)}{7}} = 1.674$, and $t = \frac{2.075-2.860}{1.674\sqrt{1/4+1/5}} = -0.70$.

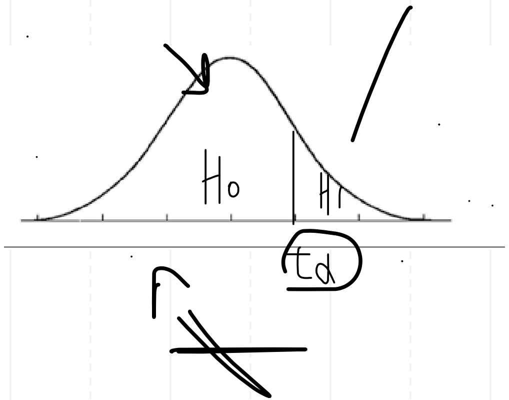
Decision: Do not reject H_0.

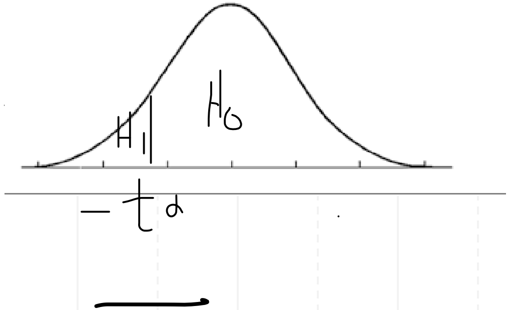
Accept H_0

Two Samples: Tests on Two Means when variances unknown but unequal

$\mu_1 - \mu_2 = d_0$	$t' = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$; $v = \dfrac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$, $\sigma_1 \neq \sigma_2$ and unknown	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t' < -t_\alpha$ $t' > t_\alpha$ $t' < -t_{\alpha/2}$ or $t' > t_{\alpha/2}$







Two Samples: Tests on Two Means when variances unknown but unequal

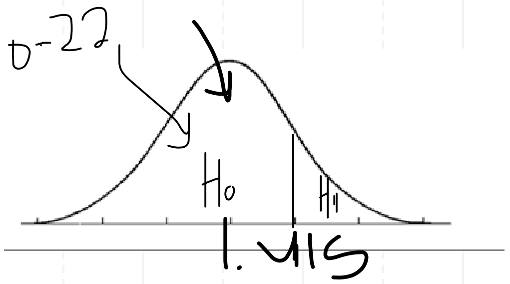
$\swarrow \delta_0$

10.39 Test $H_0: \mu 2 - \mu 1 = 10$ against $H_1: \mu 2 - \mu 1 > 10$. Use α = 0.1.

Company 1	102 86 98 109 92
Company 2	81 165 97 134 92 87 114

Assume the populations to be approximately normal with **unequal variances**.

$$H_0 : \mu_{II} - \mu_I = 10,$$
$$H_1 : \mu_{II} - \mu_I > 10.$$



$\alpha = 0.1$.

Degrees of freedom is calculated as

$$v = \frac{(78.8/5 + 913.333/7)^2}{(78.8/5)^2/4 + (913/333/7)^2/6} = 7.38,$$

hence we use 7 degrees of freedom with the critical region $t >$ 1.415

Computation: $t = \frac{(110-97.4)-10}{\sqrt{78.800/5+913.333/7}} = 0.22$.

Decision: Fail to reject H_0.

Accept H_0