كلية الحاسبات والمعلومات

المستوي الاول برنامج المعلوماتية الطبية

الفصل الدراسي الاول

2022-2021

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نموذج اجابة ورقة كاملة

المادة: تأهيلى الرياضيات

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استاذ بقسم الرياضيات بكلية العلوم ببنها

صورة من الاسئلة

Benha University
1^{st} Term (January 2022) Final Exam
Medical Informatics Program
Level: 1^{st} level
Subject: Qualifying Mathematics

Faculty of Computers & AI
Date: 10 /1 /2022
Time: 3 hrs.
Total Marks: 50 Marks
Examiner(s): Dr. Ahmed Megahed

Answer the following questions [7 questions in 5 pages]:

Q1) Find the modulus and the principle amplitude of each of the following complex

a) $z_1 = \sqrt{3} - i$ b) $z_2 = \dfrac{-1}{\sqrt{2} - \sqrt{2}i}$

Q2) Express each of the following complex numbers in trigonometric form :

$a)\ Z_1 = -2 + 2i$ $b)\ Z_2 = -4 - 4\sqrt{3}\,i$

Q3) Put each of the two numbers $\sqrt{2}\ i,\ 1+i$ in the trigonometric form then find the trigonometric form of the expression: $\left(\dfrac{\sqrt{2}\ i}{1+i}\right)^6$

Q4) Without expanding the determinant, prove that :

i) $\begin{vmatrix} 13 & 3 & 23 \\ 30 & 7 & 53 \\ 39 & 9 & 70 \end{vmatrix} = 1$ ii) $\begin{vmatrix} 3 & 5 & 6 \\ 8 & -4 & 12 \\ 4 & 7 & 5 \end{vmatrix} - \begin{vmatrix} -2 & 1 & -3 \\ 12 & 20 & 24 \\ 4 & 7 & 5 \end{vmatrix} =$ zero

Q5) Prove that $x = 3$ is one of the roots of the equation $\begin{vmatrix} x & -6 & 1 \\ 3 & -2x & x-2 \\ -2 & 3x & x+3 \end{vmatrix}$

Q6) Find the multiplicative inverse of the matrix: $A = \begin{pmatrix} 2 & 1 & -2 \\ -1 & 3 & 5 \\ 4 & 0 & 1 \end{pmatrix}$

Q7) Find the multiplicative inverse of the matrix $\begin{pmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{pmatrix}$

GOOD LUCK,

Dr. Ahmed Megahed

Model Answer

Q1)

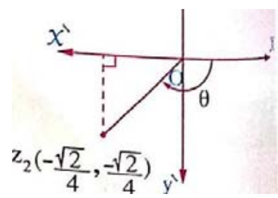
$\because z_1 = \sqrt{3} - i$ $\therefore x = \sqrt{3}, y = -1$

$\therefore |z_1| = r = \sqrt{x^2 + y^2} = \sqrt{3+1} = 2$ length unit

$, \because x > 0,\ y < 0$ $\therefore \theta$ lies in the 4^{th} quadrant

$\therefore \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$

$\therefore$ The modulus of the number $z_1 = 2$ length unit and the principle amplitude of the number $z_1 = \frac{-\pi}{6}$



$\because z_2 = \frac{-1}{\sqrt{2}-\sqrt{2}i} \times \frac{\sqrt{2}+\sqrt{2}i}{\sqrt{2}+\sqrt{2}i} = \frac{-\sqrt{2}-\sqrt{2}i}{2+2} = \frac{-\sqrt{2}-\sqrt{2}i}{4} = \frac{-\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i$

$\therefore x = \frac{-\sqrt{2}}{4}, y = \frac{-\sqrt{2}}{4}$

$\therefore |z_2| = r = \sqrt{x^2 + y^2} = \sqrt{\frac{2}{16} + \frac{2}{16}} = \frac{1}{2}$ length unit $, \because x < 0, y < 0$

$\therefore \theta$ lies in the 3^{rd} $\therefore \theta = -\pi + \tan^{-1}\left(\frac{y}{x}\right) = -\pi + \tan^{-1}\left(\frac{-\sqrt{2}}{4} \div \frac{-\sqrt{2}}{4}\right) = -\pi + \tan^{-1}(1) = 3^{rd}$
quadrant

$\therefore$ The modulus of the $\therefore$ The modulus of the number $z_2 = \frac{1}{2}$ length unit

, the principle amplitude of the number $z_2 = \frac{-3}{4}\pi$

Q2)

1. $z_1 = -2 + 2i$ $\therefore x = -2$, $y = 2$

$\therefore |z_1| = r = \sqrt{x^2 + y^2} = \sqrt{4+4} = 2\sqrt{2}$

, $\because x < 0$, $y > 0$ $\therefore \theta$ lies in the 2nd quadrant

$\therefore \theta = \pi + \tan^{-1}\left(\frac{y}{x}\right) = \pi + \tan^{-1}\left(\frac{2}{-2}\right) = \pi + \tan^{-1}(-1) = \pi + \left(\frac{-1}{4}\pi\right) = \frac{3}{4}\pi$

, $\because z_1 = r(\cos\theta + i\sin\theta)$ $\therefore z_1 = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

2. $z_2 = -4 - 4\sqrt{3}\,i$ $\therefore x = -4$, $y = -4\sqrt{3}$

$\therefore |z_2| = r = \sqrt{x^2 + y^2} = \sqrt{16+48} = 8$

$\because x < 0$, $y < 0$ $\therefore \theta$ lies in the 3rd quadrant

$\therefore \theta = -\pi + \tan^{-1}\left(\frac{y}{x}\right) = -\pi + \tan^{-1}\left(\frac{-4\sqrt{3}}{-4}\right)$

$= -\pi + \tan^{-1}\left(\sqrt{3}\right) = -\pi + \frac{\pi}{3} = \frac{-2\pi}{3}$

, $\because z_2 = r(\cos\theta + i\sin\theta)$

$\therefore z_2 = 8\left(\cos\left(\frac{-2\pi}{3}\right) + i\sin\left(\frac{-2\pi}{3}\right)\right)$

Q3)

$\because i = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ $\therefore \sqrt{2}i = \sqrt{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

, let $z = 1 + i$, $\therefore x = 1, y = 1$

$\therefore r = |z| = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}$

, $\because x > 0,\ y > 0$ $\therefore$ z lies in the 1st quadrant

$\therefore \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(1) = \frac{\pi}{4}$ $\therefore 1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\therefore \frac{\sqrt{2}i}{1+i} = \frac{\sqrt{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)} = \cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$ $\therefore \frac{\sqrt{2}i}{1+i} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

Q4)

i)

$$= \begin{vmatrix} 13 & 3 & 23 \\ 30 & 7 & 53 \\ 39 & 9 & 70 \end{vmatrix} \left(R_1 \times (-3) + R_3\right) = \begin{vmatrix} 13 & 3 & 23 \\ 30 & 7 & 53 \\ 0 & 0 & 1 \end{vmatrix} \left(C_2 \times (-4) + C_1\right)$$

$$= \begin{vmatrix} 1 & 3 & 23 \\ 2 & 7 & 53 \\ 0 & 0 & 1 \end{vmatrix} \left(R_1 \times (-2) + R_2\right)$$

$$= \begin{vmatrix} 1 & 3 & 23 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{vmatrix}$$ (The determinant in the triangular from) $= 1 \times 1 \times 1 = 1 =$ R.H

ii)

$$\text{L.H.S.} = \begin{vmatrix} 3 & 5 & 6 \\ 8 & -4 & 12 \\ 4 & 7 & 5 \end{vmatrix} - \begin{vmatrix} -2 & 1 & -3 \\ 12 & 20 & 24 \\ 4 & 7 & 5 \end{vmatrix}$$ (Take 4 as a common factor from R_2 of the second determinant)

$$= \begin{vmatrix} 3 & 5 & 6 \\ 8 & -4 & 12 \\ 4 & 7 & 5 \end{vmatrix} - 4 \begin{vmatrix} -2 & 1 & -3 \\ 3 & 5 & 6 \\ 4 & 7 & 5 \end{vmatrix}$$ (Interchange R_1 and R_2 of the second determinant)

$$= \begin{vmatrix} 3 & 5 & 6 \\ 8 & -4 & 12 \\ 4 & 7 & 5 \end{vmatrix} + 4 \begin{vmatrix} 3 & 5 & 6 \\ -2 & 1 & -3 \\ 4 & 7 & 5 \end{vmatrix}$$ (Multiply 4 by R_2 in the second determinant)

$$= \begin{vmatrix} 3 & 5 & 6 \\ 8 & -4 & 12 \\ 4 & 7 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 5 & 6 \\ -8 & 4 & -12 \\ 4 & 7 & 5 \end{vmatrix}$$ (Add the two determinants where $R_1 = R_3$)

$$= \begin{vmatrix} 3 & 5 & 6 \\ 0 & 0 & 0 \\ 4 & 7 & 5 \end{vmatrix}$$ (All the elements of R_2 equals zero)

$\therefore$ The determinant = zero = R.H.S.

Q5) L.H.S. $= \begin{vmatrix} x & -6 & 1 \\ 3 & -2x & x-2 \\ -2 & 3x & x+3 \end{vmatrix}$

Put $x = 3$

$\therefore$ L.H.S. $= \begin{vmatrix} 3 & -6 & 1 \\ 3 & -6 & 1 \\ -2 & 9 & 6 \end{vmatrix}$

$\because R_1 = R_2$ $\therefore$ The value of the determinant = zero $\therefore x = 3$ is one of the roots of the equation. $x = -1$

Q6)

- Finding the value of the determinant of the matrix :

$\therefore |A| = 2\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} -1 & 5 \\ 4 & 1 \end{vmatrix} - 2\begin{vmatrix} -1 & 3 \\ 4 & 0 \end{vmatrix} = 2(3-0) - 1(-1-20) - 2(0-12)$

$= 6 + 21 + 24 = 51 \neq$ zero

- Finding the cofactors matrix :

$\bar{a}_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$ $\bar{a}_{12} = -\begin{vmatrix} -1 & 5 \\ 4 & 1 \end{vmatrix} = 21$ $\bar{a}_{13} = \begin{vmatrix} -1 & 3 \\ 4 & 0 \end{vmatrix} = -12$ $\bar{a}_{21} = -\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = -1$

$\bar{a}_{22} = \begin{vmatrix} 2 & -2 \\ 4 & 1 \end{vmatrix} = 10$ $\bar{a}_{23} = -\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} = 4$, $\bar{a}_{31} = \begin{vmatrix} 1 & -2 \\ 3 & 5 \end{vmatrix} = 11$ $\bar{a}_{32} = -\begin{vmatrix} 2 & -2 \\ -1 & 5 \end{vmatrix} = -8$

, $\bar{a}_{33} = \begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} = 7$ - Findina the adjoint marrix :

∵ The matrix of the colictors $C = \begin{pmatrix} 3 & 21 & -12 \\ -1 & 10 & 4 \\ 11 & 8 & 7 \end{pmatrix}$

$\therefore \text{Adj}(A) = \begin{pmatrix} 3 & 21 & -12 \\ 1 & 10 & 4 \\ 11 & -8 & 7 \end{pmatrix}' = \begin{pmatrix} 3 & -1 & 11 \\ 21 & 10 & 8 \\ -12 & 4 & 7 \end{pmatrix}$

- Finding A^{-1} :

$$\because A^{-1} = \frac{1}{|\cdot A|}\text{Adj}(A) = \frac{1}{51}\begin{pmatrix} 3 & -1 & 11 \\ 21 & 10 & -8 \\ -12 & 4 & 7 \end{pmatrix} = \begin{pmatrix} \frac{1}{17} & \frac{-1}{51} & \frac{11}{51} \\ \frac{7}{17} & \frac{10}{51} & \frac{-8}{51} \\ \frac{-4}{17} & \frac{4}{51} & \frac{7}{51} \end{pmatrix}$$

Q7)

$$A = x\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \therefore A = xI$$

$$\therefore \; A^{-1} = (XI)^{-1} = \frac{1}{X} I$$

$$\therefore A^{-1} = \frac{1}{x}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Dr. Ahmed Mostafa Megahed