Faculty of Computers & Artificial Intelligence, Benha University

Student Name: ……………………………………………………

Seat Number: ……………………………………………………

Academic Year: ………….. / …………..

☐ First Semester ☐ Second Semester ☐ Summer

Program Name: ………………………………………

Course Name: ……………………………………………………

Exam Date: ……./……./ …………..

Question No	Marks attained	Full Mark	Examiner
Q1		10	
Q2		10	
Q3		10	
Q4		10	
Q5		10	
Q6			
Q7			
Q8			
Q9			
Q10			
Total For written exam		50	
Class Work			
TOTAL MARKS		50	

Total Marks

100

Total Marks (in Letters)	………………………………………………………………………………..		
Examination Committee	Examiner No. 1	Examiner No. 2	Examiner No. 3

Faculty of Computers & Artificial Intelligence
1st Term (January 2020) Final Exam
Medical Informatics Program
Course Code: MBS001 **Level:** 1st level
Subject: Qualifying Mathematics

Benha University
Date: 2 / 1 /2020
Time: 3 Hours
Total Marks: 50 Marks
Examiner(s): Associate **Prof.** E. M. Badr

Answer the following questions [5 questions in 2 pages]:

Question No. 1 [10 Marks]

Choose the correct answer:

1- $\begin{pmatrix} 4 & 5 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 7 \\ 8 \end{pmatrix}$ =....................................

a) $\begin{pmatrix} 68 \\ 17 \end{pmatrix}$ b) There is no answer c) $\begin{pmatrix} 68 & 17 \end{pmatrix}$

2- The inverse of the matrix $\begin{pmatrix} 9 & 5 \\ 2 & 8 \\ 5 & 6 \end{pmatrix}$ is ....

a) $\begin{pmatrix} 9 & 2 & 5 \\ 5 & 8 & 6 \end{pmatrix}$ b) There is no answer c) $\begin{pmatrix} 8 & -5 \\ -2 & 9 \\ 5 & 6 \end{pmatrix}$

3- The value of the determinant $\begin{vmatrix} 0 & 0 & 3 \\ 0 & 4 & 27 \\ -2 & 9 & 15 \end{vmatrix}$ is ....

a) -24 b) There is no answer c) 24

4- The center of the sphere $x^2 - 6x + 27 + y^2 - 6y + z^2 - 6z = 9$ is ....

a) (3, 3, 3) b) There is no answer (1, 2, 3)

5- The radius of the sphere $x^2 - 6x + y^2 - 6y + z^2 - 6z = -18$ is ....

a) 3 b) 6 c) 2

6- The Euler's formula of the complex number $\frac{1}{1+i}$ is ....

a) $\frac{1}{\sqrt{2}} e^{-(\frac{\pi}{4} i)}$ b) $e^{\frac{\pi}{4} i}$ c) $e^{-(\frac{\pi}{4} i)}$

7- The conjugate number of the complex number 7 is ………………………………

a) 7 b) -7 c) 0

8- If the polar representation of the point P is $(5,150^o)$ then the Cartesian representation of the point P is ………………

a) $P(\frac{-5}{2}\sqrt{3},\frac{5}{2})$ b) $P(\frac{5}{2}\sqrt{3},\frac{5}{2})$ c) $P(\frac{5}{2}\sqrt{3},\frac{2}{5})$

9- If z = -2 + 2i the argument (z) = ………………

a) $\frac{3}{4}\pi$ b) $\frac{-1}{4}\pi$ c) $\frac{-3}{4}\pi$

10-The rank of the matrix $\begin{pmatrix} 2 & 3 & -1 \\ 5 & 7 & 9 \end{pmatrix}$ is ……………………

a) 2 b) 3 c) 1

Question No. 2 [10 Marks]

i) Prove that: $x = 3$ is one of the roots of the equation: $\begin{vmatrix} x & -6 & 1 \\ 3 & -2x & x-2 \\ -2 & 3x & x+3 \end{vmatrix} = 0$

Solution:

$$L.H.S = \begin{vmatrix} x & -6 & 1 \\ 3 & -2x & x-2 \\ -2 & 3x & x+3 \end{vmatrix}$$

Put x =3, we have

$$L.H.S = \begin{vmatrix} 3 & -6 & 1 \\ 3 & -6 & 1 \\ -2 & 9 & 6 \end{vmatrix}$$

Since R1 = R2, then the value of the determinante equales to zero
So L.H.S =R. H .S.

Therefor x =3 is the root of the equation.

ii) Without expanding the determinant, Prove that: $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

Solution:

$$L.H.S = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$ **(take abc as a common factor from C3)**

$$L.H.S = abc \begin{vmatrix} 1 & a & 1/a \\ 1 & b & 1/b \\ 1 & c & 1/c \end{vmatrix}$$ **(R1 *a, R2*b and R3*c)**

$$L.H.S = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$ **(Interchange C2 and C3 then C1 and C2)**

$$L.H.S = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$ **(Interchange the rows and the columns)**

$$L.H.S = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = R.H.S$$

Question No. 3 [10 Marks]

i) If $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & -4 \\ 4 & 3 & -2 \end{pmatrix}$ **find Adj(A)**

Solution:

Cofactor (A) = $$\begin{pmatrix} \begin{vmatrix} -1 & -4 \\ 3 & -2 \end{vmatrix} & -\begin{vmatrix} 2 & -4 \\ 4 & -2 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix} \\ -\begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 1 & -3 \\ 4 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} \\ \begin{vmatrix} 2 & -3 \\ -1 & -4 \end{vmatrix} & -\begin{vmatrix} 1 & -3 \\ 2 & -4 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 14 & -12 & 10 \\ -5 & 10 & 5 \\ -11 & -2 & -5 \end{pmatrix}$$

Adj(A)=(Cofactor (A))^T = $\begin{pmatrix} 14 & -12 & 10 \\ -5 & 10 & 5 \\ -11 & -2 & -5 \end{pmatrix}^T$

ii) Solve the following system of linear equations:

$x + 2y - 3z = 6; \quad 2x - y - 4z = 2; \quad 4x + 3y - 2z = 14$

Solution:

$\begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & -4 \\ 4 & 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 14 \end{pmatrix}$ **then** $|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & -4 \\ 4 & 3 & -2 \end{vmatrix} = -40 \neq 0$

Adj(A) = $\begin{pmatrix} 14 & -12 & 10 \\ -5 & 10 & 5 \\ -11 & -2 & -5 \end{pmatrix}^T = \begin{pmatrix} 14 & -5 & -11 \\ -12 & 10 & -2 \\ 10 & 5 & -5 \end{pmatrix}$

$A^{-1} = \frac{1}{-40}\begin{pmatrix} 14 & -5 & -11 \\ -12 & 10 & -2 \\ 10 & 5 & -5 \end{pmatrix}$ **then**

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-40}\begin{pmatrix} 14 & -5 & -11 \\ -12 & 10 & -2 \\ 10 & 5 & -5 \end{pmatrix}\begin{pmatrix} 6 \\ 2 \\ 14 \end{pmatrix} = \frac{1}{-40}\begin{pmatrix} -80 \\ -80 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

Question No. 4 [10 Marks]

i) Determine the center and radius length of the sphere whose equation is:

$$x^2 + y^2 + z^2 + 2x - 4y + 6z + 12 = 0$$

Solution:

From the general form of sphere we have the center is (-1, 2, -3) and the radius length is

$r = \sqrt{(-1)^2 + (2)^2 + (-3)^2 - 12} = \sqrt{2}$

ii) If the two spheres:

$$(x-3)^2+(y-1)^2+(z-2)^2=100; \quad (x+1)^2+(y-4)^2+(z-k)^2=9$$

are touching each other, find the value of *k*?

Solution:

Center of the first sphere M1 = (3, 1, 2) and its radius is 10.

Center of the first sphere M1 = (-1, 4, k) and its radius is 10.

$$M1M2=\sqrt{(3+1)^2+(1-4)^2+(2-K)^2}=\sqrt{25+(2-K)^2}$$

First case: touching external

M1M2=10+3=13

$$\sqrt{25+(2-K)^2}=13$$

$$25+(2-K)^2=169$$

$$(2-K)^2=169$$

$$4-4k+k^2=169$$

$$k=14 \, or \quad k=-10$$

Second case: touching internal

M1M2=10-3=7

$$\sqrt{25+(2-K)^2}=7$$

$$25+(2-K)^2=49$$

$$(2-K)^2=49$$

$$4-4k+k^2=49$$

$$k=2+2\sqrt{6} \, or \quad k=2-2\sqrt{6}$$

Question No. 5 [10 Marks]

i) **If** $z=\dfrac{(2+i)(8+4i)}{(4+2i)}$ **, find |z| and arg(z) ?**

Solution:

$$z=\frac{(2+i)(8+4i)}{(4+2i)}=4+2i$$

$$|z|=r=\sqrt{4^2+2^2}=\sqrt{20}=2\sqrt{5}$$

$$\theta = \tan^{-1}\frac{2}{4} = \tan^{-1}\frac{1}{2} = 26^{o}54^{'}18^{''}$$

ii) If $z = 2 + 2\sqrt{3}\, i$ **, find** z^{4} **?**

Solution:

$$|z| = r = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{16} = 4$$

$$\theta = \mathrm{an}^{-1}\sqrt{3} = 60$$

$$z = 4(\cos 60 + i \sin 60)$$

$$z^{4} = [4(\cos 60 + i \sin 60)]^{4}$$

$$z^{4} = 4^{4}(\cos 4*60 + i \sin 4*60)$$

$$z^{4} = 256(\cos 240 + i \sin 240)$$

$$z^{4} = 256(\cos -120 + i \sin -120)$$